

# BLG theory with generalized Jordan triple systems

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## Abstract

We use a generalized Jordan algebra of the second kind to study the recently proposed BLG theory of multiple M2-branes. We find the restriction imposed on the ternary product from its consistency with the BLG theory.

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The various string theories are believed to unify into the eleven-dimensional M-theory, based on strong circumstantial evidences. M-theory contains two types of configurations in its spectrum, namely membranes and pentabranes, M2- and M5-branes, respectively, for short. These are envisaged as strong-coupling manifestations of their “stringy” counterparts, namely, D2- and D5-branes, respectively. However, while the world-volume theories of D-branes have been studied extensively from various points of view, leading occasionally to previously unknown field theories, a world-volume description of M-branes have been lacking. Recently, the world-volume theory of coalescing M2-branes have received a lot of attention. The approaches to obtaining a theory of M2-branes may be broadly classified in two categories. The first is based solely on supersymmetry considerations and a modification of the algebraic structure of the underlying gauge theory, thereby fixing all interactions [1–5]. The other is modelled after quiver gauge theories cashing in on the AdS-CFT correspondence [6]. It is not unlikely that these two approaches are related too. In this article we shall pursue the first approach, working with a BLG theory, named after its inventors [1, 5].

Three attributes are taken to be the hallmark of the world-volume theory of multiple M2-branes. First, it is to have eight bosonic scalar fields which may be interpreted as the eight directions transverse to the M2-branes in the eleven-dimensional M-theory. secondly, one expects the theory to have sixteen supersymmetries, as in the theory of D2-branes. Thirdly, it is to emerge at the conformally invariant infra-red fixed point of the theory of coalescing D2-branes. The BLG proposal satisfies these criteria. The field content of the BLG theory are eight bosonic scalars, fermions and gauge fields valued in a ternary algebra. Let us remark in passing that the use of ternary or  $n$ -ary algebras for modelling configurations in a strongly coupled situation is not unprecedented. Such algebras have been used in the context of hadronic Physics earlier [7, 8]. A ternary algebra was used in such instances also to model three-body interactions. While M2-branes are configurations obtained from D2-branes in a strongly coupled regime, the algebra in this case is not used to describe three-body interactions. On the contrary, it is deemed to model BPS configuration of multiple M2-branes. The use of a ternary algebra in this context as opposed to an  $n$ -ary one for  $n > 3$  is motivated by supergravity considerations [9].

Various aspects of the proposed theory based on a ternary algebra has been worked out [10–22]. One of the hurdles in the construction of the theory seems to be a choice of the ternary algebra itself. If the algebra is characterized by completely anti-symmetric structure constants satisfying a certain identity, called the fundamental identity, which generalizes the Jacobi identity of Lie algebras, and is required to have a Euclidean metric made out of bilinear invariants of the algebra, then the only choice for structure constants one is left with is proportional to the rank four antisymmetric tensor. One can therefore only recover the theory of *two* D2-branes from it [10, 11]. Interesting variations of this

theory by relaxing one or more of these requirements have been considered, but a consensus on *the* theory of multiple M2-branes is being awaited. In this article we propose to use a generalized Jordan triple system of the second kind (GJTS-II) [23, 24], which is a ternary algebra which does not use the fundamental identity.

A GJTS-II is an algebra  $\mathcal{A} = \{V, \langle \rangle\}$  consisting of a finite-dimensional vector space  $V$ , over a field of characteristic zero, chosen to be the field of real numbers here, endowed with a ternary product

$$\begin{aligned} V \otimes V \otimes V &\longrightarrow V, \\ (A, B, C) &\mapsto \langle ABC \rangle, \end{aligned} \tag{1}$$

obeying the identities

$$\langle AB \langle CDE \rangle \rangle - \langle \langle ABC \rangle DE \rangle + \langle C \langle BAD \rangle E \rangle - \langle CD \langle ABE \rangle \rangle = 0, \tag{2}$$

$$\begin{aligned} \langle AE \langle CBD \rangle \rangle - \langle \langle CBD \rangle EA \rangle + \langle AB \langle CED \rangle \rangle + \langle C \langle BAE \rangle D \rangle \\ - \langle AE \langle DBC \rangle \rangle + \langle \langle DBC \rangle EA \rangle - \langle AB \langle DEC \rangle \rangle - \langle D \langle BAE \rangle C \rangle = 0, \end{aligned} \tag{3}$$

where  $A, B, C, D, E$  are elements of  $V$ . The identities (2) and (3) are referred to, respectively, as Jacobson's and Kantor's identities. These identities guarantee the emergence of graded Lie algebras from a GJTS-II. Indeed, *all* compact and non-compact semi-simple Lie algebras can be recovered from different classes of GJTS-II upon choosing different vector spaces  $V$  and appropriate ternary products [25]. The linear endomorphism of  $\mathcal{A}$ ,

$$L_{AB}(C) := \langle ABC \rangle, \tag{4}$$

satisfying

$$[L_{AB}, L_{CD}] = L_{L_{AB}(C), D} - L_{C, L_{BA}(D)}, \tag{5}$$

by the Jacobson's identity (2), defines a derivation on the algebra  $\mathcal{A}$  as

$$\mathcal{D}_{AB} := L_{AB} - L_{BA}, \tag{6}$$

such that

$$[\mathcal{D}_{AB}, L_{CD}] = L_{\mathcal{D}_{AB}(C), D} + L_{C, \mathcal{D}_{AB}(D)}, \tag{7}$$

as can be verified using (5). Let us remark that, had we required the endomorphism  $L$  to be a derivation with respect to the ternary product, then the corresponding Leibniz rule

$$L_{AB} \langle CDE \rangle = \langle L_{AB}(C) DE \rangle + \langle CL_{AB}(D) E \rangle + \langle CD L_{AB}(E) \rangle \tag{8}$$

would have become the so-called “fundamental identity”,

$$\langle AB \langle CDE \rangle \rangle = \langle \langle ABC \rangle DE \rangle + \langle C \langle ABD \rangle E \rangle + \langle CD \langle ABE \rangle \rangle. \tag{9}$$

This feature is different from the “usual” gauge theories based on Lie algebras, as well as the traditional version of the BLG theory, where the gauge and matter fields belong to the same Lie triple system. We do not require the ternary product to satisfy the fundamental identity, unlike the traditional BLG theory. Indeed, combining the algebra  $\mathcal{A}$  and the derivations  $\mathcal{D}$  together in a single set, one obtains a Lie triple system from the GJTS-II, and thence a graded Lie algebra, usually referred to as a Kantor algebra, with the identities (2) and (3) enforcing the Jacobi identity of the generators of the Lie algebra [23, 25].

A bilinear form, attributed to Yamaguti [26], on a GJTS-II is defined as [23, 24]

$$\gamma(A, B) := \frac{1}{2} \text{Tr} (2R_{AB} + 2R_{BA} - L_{AB} - L_{BA}), \quad (10)$$

where the linear transformations  $L$  and  $R$  are defined as  $L_{AB}(C) = \langle ABC \rangle$  and  $R_{AB}(C) = \langle CAB \rangle$ , and the trace is taken over the linear endomorphisms. The bilinear form  $\gamma$ , known as the *trace form*, coincides with the Killing form of the corresponding Kantor algebra, up to normalization, provided  $\mathcal{A}$  satisfies some extra condition.

For future use let us introduce the fully antisymmetrized combination of the ternary brackets

$$\Omega_{AB}(C) = \langle ABC \rangle + \langle BCA \rangle + \langle CAB \rangle - \langle BAC \rangle - \langle ACB \rangle - \langle CBA \rangle \quad (11)$$

$A, B, C$  being vectors in  $V$ . This satisfies  $\Omega_{AB}(C) = \Omega_{BC}(A) = \Omega_{CA}(B)$  and  $\Omega_{AB}(C) = -\Omega_{BA}(C)$ . We shall, whenever convenient, use a set of basis vectors of  $V$  and denote them by  $\tau$ , so that  $V = \{\tau_a\}_{\text{span}}$ , with the subscript  $a$  belonging to an appropriate index set on  $\mathcal{A}$ . We shall often abuse notation by labelling operators with the indices  $a, b$  and suppressing  $\tau$ . For example, we shall use the symbols  $L_{ab}$  and  $\mathcal{D}_{ab}$  for  $L_{\tau_a \tau_b}$  and  $\mathcal{D}_{\tau_a \tau_b}$ , respectively.

Let us now consider the BLG theory in terms of a GJTS-II, beginning with the field content of the three-dimensional world-volume theory of M2-branes. The world-volume fills the directions 0, 1, 2, which will be indicated by Greek letters. The transverse directions furnish eight scalars to the world-volume gauge theory,  $X^I$ ,  $I = 3, \dots, 10$ . Correspondingly, there are eight  $Spin(1, 2)$  fermions in the world-volume collected together in a spinor field, denoted  $\psi$ . Thus,  $\psi$  is an eleven-dimensional Majorana spinor with sixteen independent real components satisfying  $\Gamma_{012}\psi = -\psi$ . In addition, the closure of the supersymmetry algebra calls for the incorporation of a vector field, which is required to be non-dynamical [27] and hence chosen to appear in the action as a Chern-Simons term only. We assume that the bosons  $X^I$  as well as the fermion  $\psi$  are valued in the vector space  $V$ . Thus, with the assumption that  $V$  to be the linear span of a finite number of

vectors  $\tau$ , as mentioned above, we write the fields as

$$\begin{aligned} X^I &= \sum_a x_a^I \tau_a \\ \psi &= \sum_a \psi_a \tau_a. \end{aligned} \tag{12}$$

The vector fields are taken to be valued in  $Der(\mathcal{A})$  as

$$A_\mu = \sum_{a,b} A_\mu^{ab} \mathcal{D}_{ab}. \tag{13}$$

It follows from (6) that  $A_\mu^{ab}$  is antisymmetric in the indices  $a, b$ . We shall find that  $A_\mu$  plays the role of gauge fields in the sequel. The BLG action is written in terms of the bilinear trace form (10) as

$$\begin{aligned} \mathcal{L} = \mathcal{L}_{\text{CS}} - \frac{1}{2} \gamma(D_\mu X^I, D^\mu X^I) + \frac{i}{2} \gamma(\bar{\psi}, \gamma^\mu D_\mu \psi) - \frac{i}{4} \Gamma^{IJ} \gamma(\bar{\psi}, \Omega_{X^I X^J}(\psi)) \\ - \frac{1}{12} \gamma(\Omega_{X^J X^K}(X^I), (\Omega_{X^J X^K}(X^I))), \end{aligned} \tag{14}$$

where  $\mu = 0, 1, 2$  is an index on the three-dimensional world-volume,  $\gamma^\mu$  are the three-dimensional Gamma matrices,  $\Gamma^I$  are the eight-dimensional Gamma matrices. We defined the derivation  $D_\mu$  on the fields as

$$D_\mu \Phi = \partial_\mu \Phi - A_\mu(\Phi), \tag{15}$$

with the action of  $A_\mu$  on a field  $\Phi$  defined in accordance with (13) as

$$A_\mu(\Phi) = \sum_{a,b,c} A_\mu^{ab} \Phi^c \langle \tau_a \tau_b \tau_c \rangle. \tag{16}$$

We shall find that  $D_\mu$  acts as a covariant derivative for a certain gauge transformation of the fields and we shall assign a transformation of  $A_\mu^{ab}$  under the gauge transformation, as we come across it in the sequel. For the time being it suffices to treat it as a vector field in the theory. The Chern-Simons term is defined as in the traditional BLG theory [1]

$$\mathcal{L}_{\text{CS}} = \frac{1}{2} \text{Tr } A \wedge F, \tag{17}$$

where we defined  $F_{\mu\nu} = [D_\mu, D_\nu]$  and the trace is as in (10). The action  $\mathcal{L}$  in (14) is invariant under the supersymmetry transformations [3],

$$\delta X^I = i\bar{\theta} \Gamma^I \psi \tag{18}$$

$$\delta \psi = D_\mu X^I \gamma^\mu \Gamma^I \theta + \langle X^I X^J X^K \rangle \Gamma^{IJK} \theta \tag{19}$$

$$\delta A_\mu(\Phi) = i\bar{\theta} \gamma_\mu \Gamma^I \Omega_{\psi X^I}(\Phi), \tag{20}$$

where  $\Phi$  is either of the bosonic or fermionic fields,  $X^I$  or  $\psi$ . Let us note that (19) can be re-written in terms of  $\Omega$  as

$$\delta\psi = D_\mu X^I \gamma^\mu \Gamma^I \theta + \frac{1}{3!} \Omega_{X^I X^J} (X^K) \Gamma^{IJK} \theta. \quad (21)$$

The expression on the right hand side of (20) requires qualification in the case when  $\Phi$  is the fermionic field. In a product of two fermions  $\bar{\psi}$  must appear on the left of  $\psi$ , in order to combine into a scalar in the fermionic indices. Thus, when  $\Phi = \psi$ , we need to arrange the fermions on the right hand side of (20) such that the fermions are in proper order. Accordingly, in this case, using the following expressions

$$\begin{aligned} \langle (\bar{\theta} \gamma_\mu \Gamma^I \psi) X (\bar{\theta} \Gamma^J \psi) \rangle &= -\langle (\bar{\psi} \gamma_\mu \Gamma^I \theta) X (\bar{\theta} \Gamma^J \psi) \rangle \\ \langle (\bar{\theta} \Gamma^I \psi) X (\bar{\theta} \gamma_\mu \Gamma^J \psi) \rangle &= -\langle (\bar{\psi} \Gamma^I \theta) X (\bar{\theta} \gamma_\mu \Gamma^J \psi) \rangle \end{aligned} \quad (22)$$

whenever necessary, the expression on the right hand side of (20) becomes

$$\delta A_\mu(\psi) = -i M_\nu(\psi) \gamma^\nu \gamma_\mu \Gamma^I \theta, \quad (23)$$

where

$$M_\nu(\Phi) = \langle \bar{\psi} \gamma_\nu \psi \Phi \rangle - \langle \bar{\psi} \Phi \gamma_\nu \psi \rangle + \langle \Phi \bar{\psi} \gamma_\nu \psi \rangle. \quad (24)$$

The Fierz identities,

$$(\bar{\psi} \chi) \lambda = -\frac{1}{2} (\bar{\psi} \lambda) \chi - \frac{1}{2} (\bar{\psi} \gamma_\mu \lambda) \gamma^\mu \chi \quad (25)$$

$$\lambda (\bar{\psi} \chi) = -\frac{1}{2} \psi (\bar{\lambda} \chi) + \frac{1}{2} \gamma_\mu \psi (\bar{\lambda} \gamma^\mu \chi) \quad (26)$$

where  $\psi$ ,  $\chi$  and  $\lambda$  are sixteen-dimensional Majorana spinors, were used to prove the supersymmetry-invariance of the action (14).

Commutators of the supersymmetry transformations furnish consistent equations of motion, as well as the gauge transformation. For the bosonic fields,  $X$ , the commutator of two supersymmetry transformations  $\delta_1$  and  $\delta_2$  with supersymmetry parameters  $\theta_1$  and  $\theta_2$ , respectively, take the form

$$[\delta_1, \delta_2] X^I = v^\mu D_\mu X^I + \Lambda(X^I) \quad (27)$$

where

$$v^\mu = -2i \bar{\theta}_2 \gamma^\mu \theta_1 \quad (28)$$

$$\Lambda = i \bar{\theta}_2 \Gamma^{JK} \theta_1 \Omega_{X^J X^K}. \quad (29)$$

The second term in (27) is to be interpreted as a gauge transformation. Considering the most general gauge transformation of this kind, let us consider

$$\Lambda_{fg}(X) = \Omega_{fg}(X), \quad (30)$$

for arbitrary functions  $f, g$  on the world-volume. Let us note that  $\Omega$  is anti-symmetric in the lower two indices by definition, (11). Using the definition of the vector field (13) and the derivative (15), then, the latter is covariant, that is  $\Lambda_{fg}(DX) = D(\Lambda_{fg}(X))$ , if  $A_\mu^{ab}$  transforms as

$$A'_\mu(\Omega_{fg}(X)) = \frac{1}{2}(\Omega_{\partial_\mu f, g}(X) + \Omega_{f, \partial_\mu g}(X)) + \Omega_{fg}(A_\mu(X)). \quad (31)$$

The commutator acting on the fermion yields

$$\begin{aligned} [\delta_1, \delta_2]\psi &= v^\mu D_\mu \psi + \Lambda(\psi) \\ &+ i\bar{\theta}_2 \gamma^\nu \theta_1 \gamma_\nu \left( \gamma^\mu D_\mu \psi - \frac{1}{2} \Gamma^{IJ} \Omega_{X^I X^J}(\psi) \right) \\ &- \frac{i}{4} \bar{\theta}_2 \Gamma^{MN} \theta_1 \Gamma^{MN} \left( \gamma^\mu D_\mu \psi - \frac{1}{2} \Gamma^{IJ} \Omega_{X^I X^J}(\psi) \right). \end{aligned} \quad (32)$$

Closure of the supersymmetry algebra requires the braced expression in the second and the third terms on the right hand side to vanish,

$$\gamma^\mu D_\mu \psi - \frac{1}{2} \Gamma^{IJ} \Omega_{X^I X^J}(\psi) = 0. \quad (33)$$

This being the equation of motion obtained by varying  $\psi$  from (14), we conclude that the algebra closes on-shell.

Closure of the supersymmetry algebra on the vector field imposes further restriction on the ternary bracket. The gauge field acted on by the same commutator leads to terms proportional to the rank five Gamma matrix,  $\Gamma^{IJKLM}$ . The coefficient of proportionality is to vanish for the closure of the algebra. We find

$$[\delta_1, \delta_2]A_\mu(\Phi) = 2i\epsilon_{\mu\nu\lambda}(\bar{\theta}_2 \gamma^\lambda \theta_1) (\Omega_{D_\nu X^I, X^I}(\Phi) - iM_\nu(\Phi)) + D_\mu \Lambda(\Phi), \quad (34)$$

where  $\Phi$  is either a bosonic field,  $X^I$ , or the fermion,  $\psi$  and  $M_\nu(\Phi)$  is defined in (24), provided we assure the vanishing of the coefficient of  $\Gamma^{IJKLM}$  by imposing the condition

$$\begin{aligned} &[\langle \Phi X^I \langle X^J X^K X^L \rangle \rangle + \langle X^I \langle X^J X^K X^L \rangle \Phi \rangle + \langle \langle X^J X^K X^L \rangle \Phi X^I \rangle \\ &- \langle X^I \Phi \langle X^J X^K X^L \rangle \rangle - \langle \Phi \langle X^J X^K X^L \rangle X^I \rangle - \langle \langle X^J X^K X^L \rangle X^I \Phi \rangle]_{IJKL} = 0, \end{aligned} \quad (35)$$

where  $[\ ]_{IJKL}$  designates complete antisymmetrization in  $\{I, J, K, L\}$ . The constraint equation can be recast in terms of the basis vectors of  $V$  as

$$\begin{aligned} &[\langle \tau_a \tau_b \langle \tau_c \tau_d \tau_e \rangle \rangle + \langle \tau_b \langle \tau_c \tau_d \tau_e \rangle \tau_a \rangle + \langle \langle \tau_c \tau_d \tau_e \rangle \tau_a \tau_b \rangle \\ &- \langle \tau_b \tau_a \langle \tau_c \tau_d \tau_e \rangle \rangle - \langle \tau_a \langle \tau_c \tau_d \tau_e \rangle \tau_b \rangle - \langle \langle \tau_c \tau_d \tau_e \rangle \tau_b \tau_a \rangle]_{bcde} = 0. \end{aligned} \quad (36)$$

Thus, the GJTS-II yields a consistent BLG theory if and only if the algebra satisfies the constraint (36) in addition to (2) and (3). Equation (35) is the most general constraint that can be written down, irrespective of the type of ternary algebra used. One can simplify it to the case of GJTS-II, by imposing the Jacobson and Kantor identities on (35). Then the constraint takes the form

$$\begin{aligned} & [\langle \Phi \langle X^J X^K X^L \rangle X^I \rangle - \langle \Phi X^I \langle X^J X^K X^L \rangle \rangle \\ & - \langle X^I X^K \langle X^J \Phi X^L \rangle \rangle - \langle X^J \langle X^K X^I \Phi \rangle X^L \rangle ]_{IJKL} = 0. \end{aligned} \quad (37)$$

Let us point out that instead of using the GJTS-II if we assume the structure constants of  $\mathcal{A}$  to be completely antisymmetric and impose the fundamental identity, as in the traditional BLG theory, then the constraint (36) is satisfied. The present considerations generalize the BLG theory in this sense.

Continuing with the genral structure, using  $F_{\mu\nu} = [D_\mu, D_\nu]$  the supersymmetry algebra closes up to the equation of motion for  $A_\mu$ , viz.

$$F_{\mu\nu} - \epsilon_{\mu\nu\lambda} (\Omega_{D_\lambda X^I, X^I}(\Phi) + iM_\lambda(\Phi)) = 0. \quad (38)$$

As a further check on consistency, let us point out that, taking the supervariation of the equation of motion (33) we get,

$$\begin{aligned} & \Gamma^I \left( D^2 X^I + iN_{IJ}(X^J) - \frac{1}{2} \Omega_{X^J X^K}(\Omega_{X^J X^K}(X^I)) \right) \theta \\ & - \gamma^\lambda \Gamma^I \left( \frac{1}{2} \epsilon^{\mu\nu\lambda} F_{\mu\nu}(X^I) - \Omega_{D_\lambda X^J X^J}(X^I) - iM_\lambda(X^I) \right) \theta = 0. \end{aligned} \quad (39)$$

Here the second expression in braces is zero by (38). The remaining part gives the equation of motion for the bosons,

$$D^2 X^I + iN_{IJ}(X^J) - \frac{1}{2} \Omega_{X^J X^K}(\Omega_{X^J X^K}(X^I)) = 0, \quad (40)$$

where we defined

$$N_{IJ}(X^J) = \langle \bar{\psi} \Gamma^{IJ} \psi X^J \rangle + \langle X^J \bar{\psi} \Gamma^{IJ} \psi \rangle - \langle \bar{\psi} X^J \Gamma^{IJ} \psi \rangle. \quad (41)$$

In here, too, the quintic term coupled to the rank five gamma matrix cancels thanks to (2), (3) and (36).

We thus conclude that the BLG theory can be consistently written in terms of a GJTS-II, provided the algebra satisfies the constraint (35).

## Conclusion

To summarize, in this note we consider the BLG theory of multiple M2-branes based on a generalized Jordan triple system of the second kind. Unlike the prototypical formulation of the BLG theory, the ternary bracket we use is not stipulated to satisfy the so-called fundamental identity or complete antisymmetry of its structure constants and thus evades the stringent restriction on structure constants ensuing from it. Rather, the ternary product satisfies the Jacobson and Kantor identities, (2) and (3), respectively, along with an extra constraint, (36). The traditional restrictions of complete antisymmetry and fundamental identity turns out to be a special solution to the constraint equation. In this sense, our considerations generalizes the BLG proposal. Therefore, the present formulation is valid for a wide class of ternary algebras. It would be interesting to find out explicit ternary algebras that is compatible with this formulation.

So far we have interpreted the constraint (36) in the spirit of BLG as a condition on the ternary algebra itself assuming the target to be flat, which severely restrains the algebra. We can, instead, turn it around and interpret (35) as constraining the  $X$ 's themselves, without restricting the algebra. In this sense our formulation would give rise to a system of M2-branes on a non-trivial target space. In particular, considering representations of ternary algebras, it would be illuminating to examine whether a generic algebra can be incorporated in the scheme by modifying the target space appropriately.

One of the motivations for our approach here is based on the fact that a theory of multiple M2-branes, described by a ternary algebra is supposed to be a strong-coupling limit of the theory of multiple D2-branes, the latter being described, in the low-energy regime, by a Yang-Mills theory. This hints at a natural connection between a ternary algebra and a Lie algebra. Since a Jordan triple system of the second kind intrinsically leads to a graded Lie algebra, namely, the Kantor algebra, the former is a likely candidate to feature in a theory of M2-branes. However, the physical interpretation of the means of obtaining the Kantor algebra from the GJTS-II within this context is yet to be fleshed out. We hope to report on some of these issues in future.

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